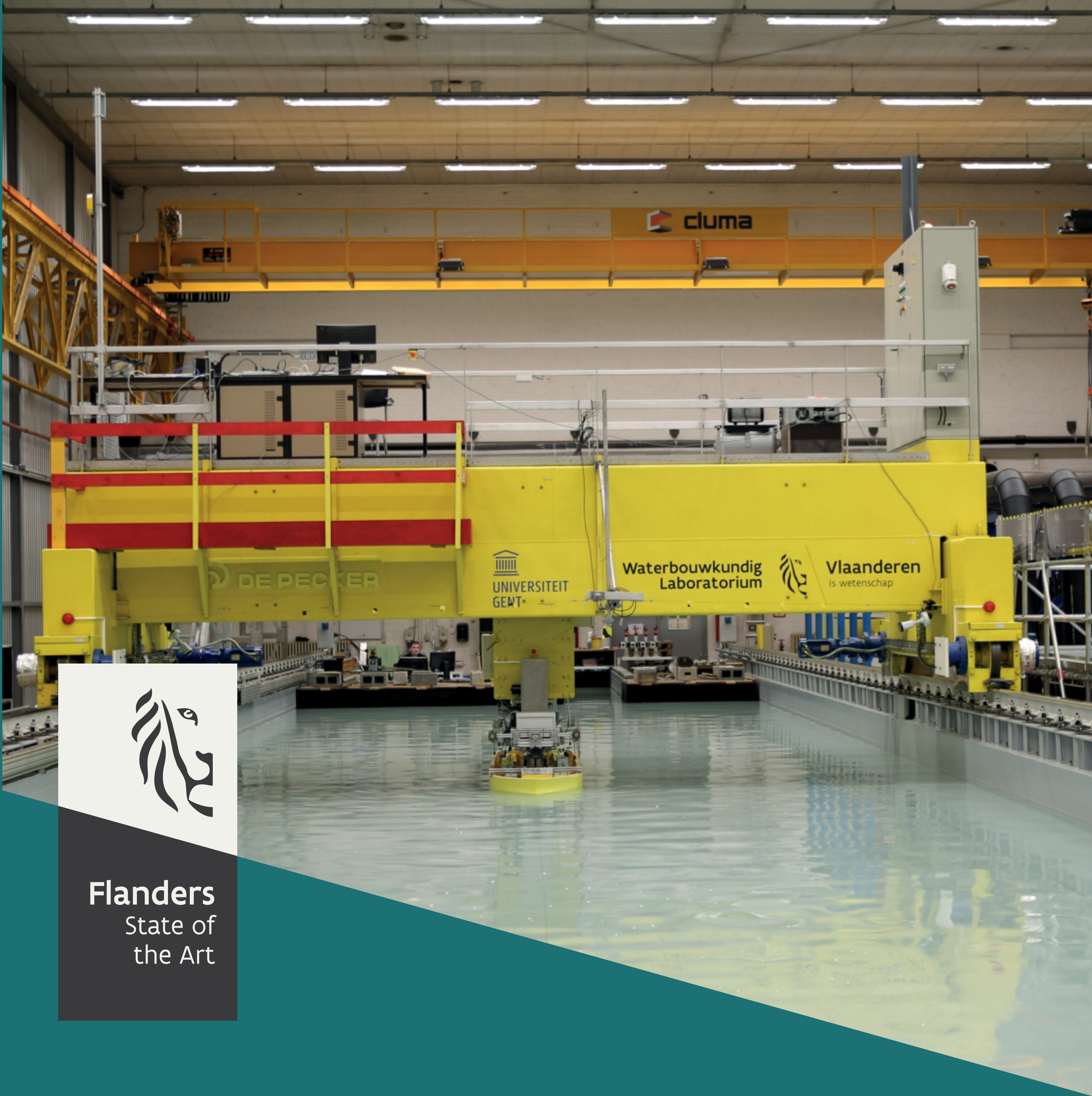

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Regstatx
Regression Software for Ship Manoeuvring Models

Sub report 3
Regstatx 4.0: theoretical background

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Regression Software for Ship Manoeuvring Models

Sub report 3 – Regstatx 4.0: theoretical background

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Abstract

This report provides a theoretical background for the in house developed software package Regstatx 4.0. Emphasis is put on the test selection process, the relationship with ODRPack 5.0 and the formulation of the 6 DOF tabular open water manoeuvring mathematical models, including assumptions, from towing tank to ship manoeuvring simulator.

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Nomenclature

Text markup:

ODRPack programs are written in italic. See the ODRPack 5.0 documentation for more information.

These numbers represent constants in Regstatx 4.0, which affect the outcome of regression analyses.

These variables are regression results.

These are concerns that may need further investigation.

Symbols list:

a_H	parameter	(-)
a_*	propeller acceleration factor	(-)
A	bifurcation angle	(deg)
A_R	rudder area	(m²)
A_W	waterline area	(m²)
B	breadth	(m)
B	bifurcation angle	(deg)
C_B	block coefficient	(-)
C_D	drag coefficient	(-)
C_L	lift coefficient	(-)
C_Q	torque coefficient	(-)
C_T	thrust coefficient	(-)
c	rudder mean chord length	(m)
D_P	propeller diameter	(m)
F	force or moment	(-)
F_N	force, perpendicular on the rudder	(N)
F_{rh}	water depth based Froude number	(-)
F_T	force, tangential on the rudder	(N)
F_X	longitudinal rudder force	(N)
F_Y	lateral rudder force	(N)
g	gravity acceleration	(m/s²)
$\overline{GM}_T$	initial transverse stability lever	(m)
$\overline{GM}_L$	initial longitudinal stability lever	(m)
h	water depth	(m)
I_{**}	moment or product of inertia	(kgm²)
$J^{(\prime)}$	(apparent) advance coefficient	(-)
K	roll moment	(Nm)
K_Q	torque coefficient	(-)
K_T	thrust coefficient	(-)
k	~ distance propeller – rudder	(-)
L	length	(m)

M	pitch moment	(Nm)
m	ship's mass	(kg)
N	yaw moment	(Nm)
n	propeller rate	(1/s)
p	roll velocity	(rad/s)
q	pitch velocity	(rad/s)
Q_P	propeller shaft torque	(Nm)
Q_R	rudder steering torque	(Nm)
r	yaw velocity	(rad/s)
T	draft	(m)
t	time (s); thrust deduction factor	(-)
T_P	propeller thrust	(N)
T_{rh}	Tuck parameter	(-)
u	longitudinal ship velocity	(m/s)
u_R	longitudinal velocity near rudder	(m/s)
ukc	under keel clearance	
V	global ship velocity	(m/s)
v	lateral ship velocity	(m/s)
v_R	lateral velocity near rudder	(m/s)
w	vertical ship velocity	(m/s)
w_Q	wake factor for the propeller torque	(-)
w_R	wake factor for the rudder	(-)
w_T	wake factor for the thrust	(-)
X	longitudinal force	(N)
x	longitudinal coordinate	(m)
x_G	longitudinal centre of gravity	(m)
x_H	parameter	(-)
x_R	longitudinal position of rudder	(m)
Y	sway force	(N)
y	lateral coordinate	(m)
y_G	lateral centre of gravity	(m)
Z	heave force	(N)
z	vertical coordinate	(m)
z_G	vertical centre of gravity	(m)
z_H	parameter	(-)
z_{HX}	parameter	(-)
z_R	vertical position rudder centreline	(m)
α	inflow angle	(deg)
β	drift angle	(deg)
β_*	regression coefficient	(-)
β_R	drift angle near rudder	(deg)
γ	yaw angle	(deg)
γ^*	propeller loading angle for yaw	(deg)

Δ	displacement	(N)
δ	rudder angle	(deg)
δ_0	rudder asymmetry correction	(deg)
$\varepsilon^{(*)}$	(apparent) propeller loading angle	(deg)
η	propeller diameter ÷ rudder height	(-)
θ	pitch angle	(deg)
μ	multiplicator	(-)
ξ	parameter	(-)
ρ	water density	(kg/m³)
φ	heel angle	(deg)
φ^*	propeller loading angle for sway	(deg)
φ_*	phase angle	(deg)
χ	yaw-drift correlation angle	(deg)
ψ^*	course angle (combination drift, yaw)	(deg)
ψ^*_{90}	course angle limited between ±90°	(deg)
ω	frequency	(rad/s)
$\partial *$	uncertainty of $*$	(-)

Subscripts

0	tank fixed
1	portside
2	starboard side
A	aft
AI	added inertia
F	fore
i	rudder, propeller number
IC	inertial and centrifugal
H	hull
hyd	hydrostatic
OA	over all
P	propeller
PP	between perpendiculars
Q	torque
R	rudder
ret	retardation
T	thrust
X	longitudinal
Y	lateral

Superscripts

.	time derivative
′	dimensionless
″	symmetric
n	propeller dependent

1 General

1.1 Programming

Regstatx 4.0.0 has been developed with Visual Studio 2015 and is written in C#. The target framework is .NET 4.5.2. The code is written from scratch and is more concise compared to Regstatx 3.0.17 (see Table 1). Some exceptional functionality from Regstatx 3.0.17. is not (yet) included (inland manoeuvring models, Z-drives, nautical bottom). The major advances are the creation of a graphical user interface, a native 6 DOF approach with two rudders and two propellers, a thorough documentation system and a full integration with the XML SOW applications.

Table 1 – Code comparison between Regstatx 3.0.17 and 4.0.0 Beta 5.9 according to Code Metrics Visual Studio 2015

	Regstatx 3.0.17	Regstatx 4.0.0 Beta 5.9			
		Data dll	Domain dll	GUI dll	Total
Maintainability Index (more is better)	38	81	74	81	74
Cyclomatic Complexity (less is better)	6,346	477	1,085	80	1,642
Depth of Inheritance (less is better)	7	4	2	10	10
Class Coupling (less is better)	153	101	86	73	260
Lines of Code (less is better)	27,766	1,229	3,415	243	4,887

Future versions will be named Regstatx 4.x.y:

- x introduces new functionalities;
- y indicates bug fixes.

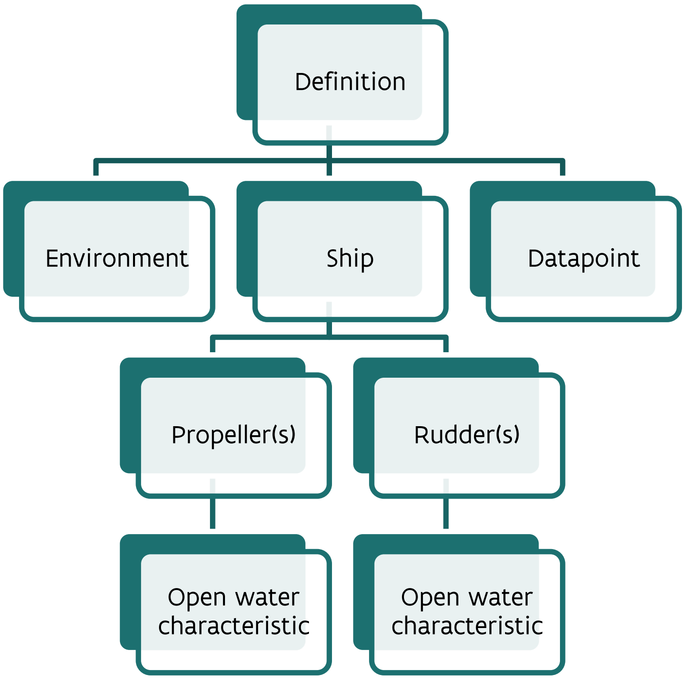
1.2 Needed files

Regstatx 4.0 makes use of a definition file to link (Figure 1):

- the test results (datapoint) with
- a ship model, composed of:
  - a hull (Ship);
  - multiple propellers, with open water characteristics;
  - multiple rudders, with open water characteristics;
- sailing in a certain environment.

The user can create his own definition file in Regstatx 4.0. All of the above mentioned files are needed before being able to carry out any regression.

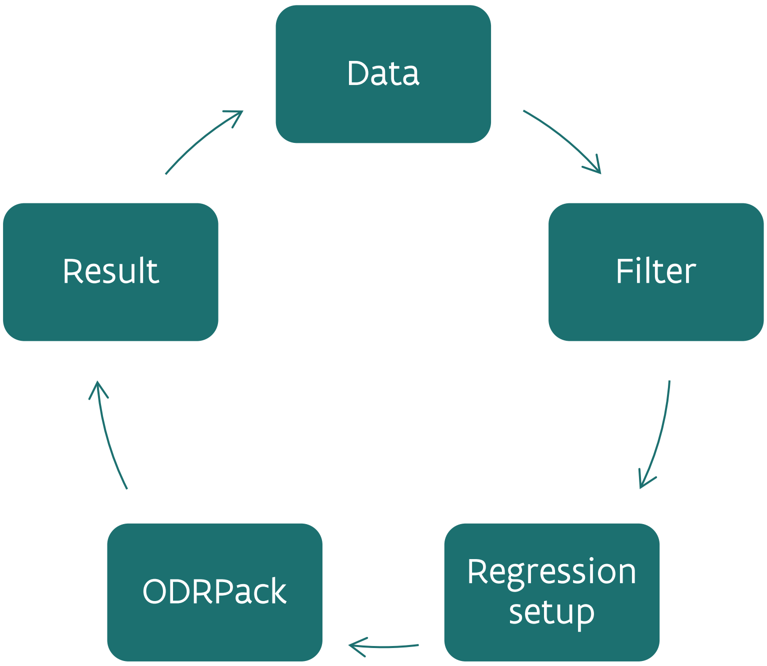
Figure 1 – Tree structure of the needed input files



1.3 Regression cycle

Regstatx 4.0 performs different iteration loops, as shown in Figure 2, to obtain a regression model.

Figure 2 – Typical computation loop in Regstatx 4.0

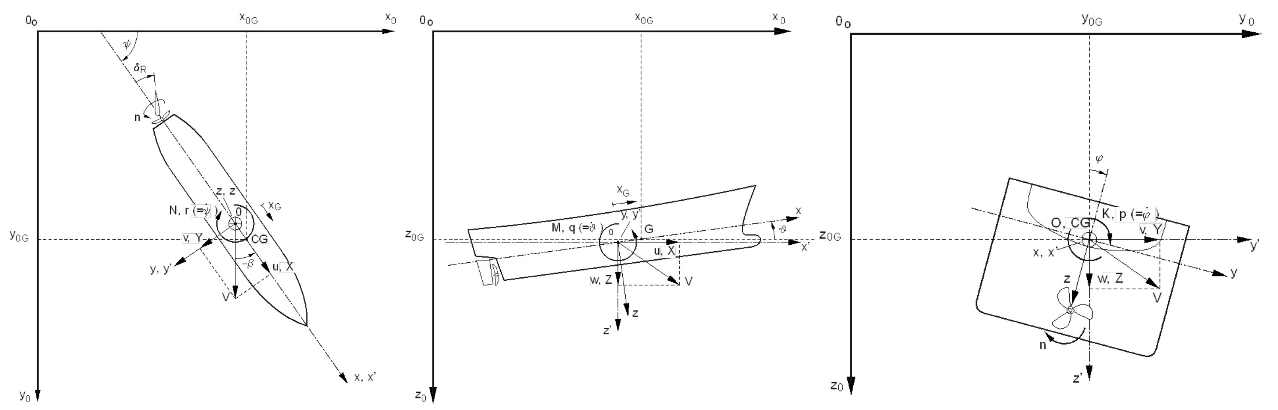


From a starting Data set (datapoint) a selection (Filter) of tests is needed. From this selection the dependent and independent variables are selected, together with needed input files and start values for the coefficients (Regression setup). This information is sent to ODRPack, which carries out the actual regression. The result is sent back to Regstatx, which performs appropriate actions, such as saving the coefficients in a correct format and updating the data, so that new regressions can be carried out.

1.4 Coordinate system

Regstatx 4.0 uses the common towing tank defined coordinate system as depicted in Figure 1. It is assumed that for the present purposes, the ship fixed coordinate system is horizontally bound.

Figure 3 – Coordinate system used in Regstatx 4.0



1.5 First computation

The first time a new definition file is read the datapoint file is updated by adding the following columns:

- None of the datapoint files created before 2017 has columns for p or $\dot{p}$. In this case zero values are automatically added for all rows.
- Inertial components of the measured forces (steering in 4 horizontal DOF):

$$X_{IC} = m[-\dot{u} + vr + x_G r^2 + y_G \dot{r} - z_G pr] \quad (1)$$

$$Y_{IC} = m[-\dot{v} - ur - x_G \dot{r} + y_G r^2 + z_G \dot{p}] \quad (2)$$

$$Z_{IC} = 0 + m[vp - x_G pr - y_G \dot{p} + z_G p^2] \quad (3)$$

$$K_{IC} = -I_{xx}\dot{p} + I_{xz}\dot{r} - I_{yx}pr - I_{yz}r^2 + m[-vpy_G + (\dot{v} + ur)z_G] \quad (4)$$

$$M_{IC} = I_{xy}\dot{p} + I_{yz}\dot{r} - I_{xz}(p^2 - r^2) - (I_{xx} - I_{zz})pr + m[vpx_G + (-\dot{u} + vr)z_G] \quad (5)$$

$$N_{IC} = -I_{zz}\dot{r} + I_{xz}\dot{p} + I_{xy}p^2 + I_{yz}pr + m[-(\dot{v} + ur)x_G + (\dot{u} - vr)y_G] \quad (6)$$

  The grey terms are neglected. Mind that the actual modelling of the roll velocity and acceleration dependent terms is not yet included in Regstatx 4.0.
- The drift angle:

$$\beta = \text{atan2}(-v, u) \quad (7)$$

- Three Tuck numbers, based on the different speed components

$$T_{rh} = \frac{F_{rh}}{\sqrt{1-F_{rh}^2}} \quad (8)$$

  with the Froude depth number defined as

$$F_{rh} = \frac{V}{\sqrt{gh}} \quad (9)$$

and the global velocity vector either

$$V = \sqrt{u^2 + v^2} \tag{10}$$

$$V = \sqrt{u^2 + \left(\frac{1}{2}\frac{r}{L}\right)^2} \tag{11}$$

$$V = \sqrt{v^2 + \left(\frac{1}{2}\frac{r}{L}\right)^2} \tag{12}$$

Only the Tuck number based on u, v is actually used in the modelling.

- The rudder forces expressed in the ship's horizontal bound coordinate system for each rudder i:

$$F_{Xi} = -F_{Ni} \sin \delta_i + F_{Ti} \cos \delta_i \tag{13}$$

$$F_{Yi} = F_{Ni} \cos \delta_i + F_{Ti} \sin \delta_i \tag{14}$$

- The total heave force (without trim/sinkage coupling)

$$Z = \frac{\rho}{1000} g A_W z \tag{15}$$

- The total pitch moment (without trim/sinkage coupling)

$$M = mg\overline{GM}_L \tan^{-1} \frac{trim}{1000} \tag{16}$$

Sinkage (and trim) are expressed in mm (per m), hence the factor 1000 in the denominators.

- A dummy column with zero values.

In the above and the following care should be taken of the correct dimensions (e.g. degrees versus radians, or revolutions per minute versus revolutions per second). In the code this is achieved by specific objects, which need to be called with their correct dimension.

1.6 Ideal computation loop

Once a definition file has been created, the ideal order of the computations is:

- Propulsion computation
- Steering computation
- Hull computation
- Propulsion induced computation
- Steering induced computation

Each of the computations will be explained with more details in the following chapters.

2 Propulsion computation

2.1 Filter

The datapoint is filtered to select the tests that have sufficient and symmetric propeller rate, a small drift and a small rudder angle.

2.1.1 Checking the propeller symmetry rate

In case the ship has two propellers, the symmetry must be checked. This is done by comparing the given propeller rates for each datapoint row i and each propeller:

$$|n_1 - n_2| < 20\ rpm \quad (17)$$

The same has to be true for the rudder angles, if the ship has two rudders equipped:

$$|\delta_1 - \delta_2| < 5° \quad (18)$$

2.1.2 Selecting tests

Tests of type STATX0, MULTI0, MULTI1 and PAAL are selected if the following conditions are fulfilled:

$$|n_1| > 0.3 n_{MAX} \quad (19)$$

$$|\delta_1| < 5° \quad (20)$$

$$|\beta| < 1° \quad (21)$$

For the multi-modal tests only tests where the ship's accelerations are equal to zero are selected.

2.2 Thrust computation for each propeller i

2.2.1 Determine the slope of the linear KT(J') characteristic in the first quadrant

2.2.1.1 Sub filter

From the filtered tests, only tests in the first quadrant are needed with sufficiently large propeller loading. Acceptable test types are STATX0, MULTI0 or MULTI1. The filtering is performed based on the apparent hydrodynamic angle:

$$0 \le \varepsilon_i^* = \text{atan2}(u, 0.7\pi n_i D_{Pi}) \le 40° \quad (22)$$

The data from bollard pull tests (PAAL) or multi-modal tests when the ship's velocities are equal to zero, are saved in a separate set. From this set the minimal and maximal thrust coefficients are computed, with the thrust coefficient defined as:

$$K_{Ti} = \frac{T_{Pi}}{\rho n_i^2 D_{Pi}^4} \quad (23)$$

2.2.1.2 Regression setup and execution

The regression is carried out with the ODRPack program *KJacc.exe*. In this routine the slope β_{Ti} is determined:

$$K_{Ti}(J'_i) = \beta_{Ti} J'_i + K_{Ti}(0) \qquad (24)$$

$K_{Ti}(0)$ is the maximal thrust coefficient measured at bollard pull conditions.

2.2.2 Update the open water characteristic of the propeller

The open water characteristic of the propeller, usually measured in deep water condition, is corrected based on the measured thrust during bollard pull condition. For a twin propeller setup it is assumed that each propeller has the same open water characteristic, consequently the correction is only carried out once.

The correction is performed per quadrant, under the assumption that the open water characteristic,

- follows a quadratic trend (Quadrants 1 and 3) between $J = 0$ and $J = J_{MAX}$;
- follows a 4th degree polynomial trend (Quadrants 2 and 4) between $J = -J_{MAX}$ and $J = 0$.

A filter groups the data of the open water characteristic per quadrant between the above described boundaries. In the program, the user can select an appropriate value for J_{MAX}. Each quadrant has its own regression program in ODRPack:

- Quadrant 1, *KJ1.exe*:

$$K_T(J) = \beta_{T11} J^2 + \beta_{T12} J + K_{T1}(0) \qquad (25)$$

  with $K_T(0)$ the maximal thrust coefficient measured at bollard pull conditions.
- Quadrant 2, *KJ2.exe*:

$$K_T(J) = \beta_{T21} J^4 + \beta_{T22} J^3 + \beta_{T23} J^2 + \beta_{T24} J + K_{T1}(0) \qquad (26)$$

  with $K_T(0)$ the minimal thrust coefficient measured at bollard pull conditions.
- Quadrant 3, *KJ3.exe*:

$$K_T(J) = \beta_{T31} J^2 + \beta_{T32} J + K_{T1}(0) \qquad (27)$$

  with $K_T(0)$ the minimal thrust coefficient measured at bollard pull conditions.
- Quadrant 4, *KJ4.exe*:

$$K_T(J) = \beta_{T41} J^4 + \beta_{T42} J^3 + \beta_{T43} J^2 + \beta_{T44} J + K_{T1}(0) \qquad (28)$$

  with $K_T(0)$ the maximal thrust coefficient measured at bollard pull conditions.

After the execution of these four programs a corrected open water characteristic $C_{Tcorr}(\varepsilon)$ is created as follows:

- Within the boundaries of the regression:

$$C_{Tcorr}(\varepsilon) = \frac{K_T(J)}{\frac{\pi}{8}(J^2 + (0.7\pi)^2)} \qquad (29)$$

  with $K_T(J)$ equal to one of the above equations.
- Outside the boundaries of the regression, the original values of $C_T(\varepsilon)$ are taken.

2.2.3 Compute the wake factors

Wake factors are computed only in the first quadrant. The following assumptions are made:

- $0 \leq w_{Ti} \leq 0.9$
- $w_{Ti}(\varepsilon^* \geq 30°) = w_{Ti}(\varepsilon^* = 30°)$

Additionally the user has the opportunity to ensure a smooth transition between the first and the fourth quadrant, by allowing a linear decrease of the wake factor between $-90° \leq \varepsilon^*{}_{user} \leq 0°$ and 0°, instead of a sudden drop to zero.

The wake factors are determined based on the thrust identity:

$$K_{Ti}(J_i') = \beta_{Ti}J_i' + K_{Ti}(0) \equiv K_T(J) = \beta_{T11}J^2 + \beta_{T12}J + K_{T1}(0) \tag{30}$$

$$J_i' = \frac{\beta_{T11}J^2+\beta_{T12}J}{\beta_{Ti}} \tag{31}$$ [1]

and

$$w_{Ti} = \frac{J_i'-J}{J_i'} = \frac{\beta_{T11}J^2+(\beta_{T12}-\beta_{Ti})J}{\beta_{T11}J^2+\beta_{T12}J} \tag{32}$$

From the above equations it is fairly easy to write w as a function of J. In practice J_i' is the known parameter and J has to be found, however, until now it has been assumed that $w(J_i') = w(J)$ as the thrust is sufficiently well predicted. In the model outcome w is expressed as a function of ε_i^*:

$$J_i' = 0.7\pi \tan \varepsilon_i^* \tag{33}$$

In bollard pull conditions the wake factor is defined as:

$$\lim_{J_i' \to 0} \frac{J_i'-J}{J_i'} \tag{34}$$

In practice, the value from the next wake factor in the $w_{Ti}(\varepsilon^*)$ table is taken. In normal cases β_{Ti} is strictly negative. In case a positive value is obtained, w_{Ti} is equal to 0.9 in the entire first quadrant.

The uncertainty of the wake factor is dependent of the uncertainty of the individual regression coefficients:

$$\partial w_{Ti} = \sqrt{\begin{gathered}\left[\frac{J^2(\beta_{T11}J^2+\beta_{T12}J)-J^2(\beta_{T11}J^2+(\beta_{T12}-\beta_{Ti})J)}{(\beta_{T11}J^2+\beta_{T12}J)^2}\partial\beta_{T11}\right]^2 + \\ \left[\frac{J(\beta_{T11}J^2+\beta_{T12}J)-J(\beta_{T11}J^2+(\beta_{T12}-\beta_{Ti})J)}{(\beta_{T11}J^2+\beta_{T12}J)^2}\partial\beta_{T12}\right]^2 + \left(-\frac{J}{\beta_{T11}J^2+\beta_{T12}J}\partial\beta_{Ti}\right)^2\end{gathered}} \tag{35}$$

which simplifies to:

$$\partial w_{Ti} = \sqrt{\left[\frac{J^3\beta_{Ti}}{(\beta_{T11}J^2+\beta_{T12}J)^2}\partial\beta_{T11}\right]^2 + \left[\frac{J^2\beta_{Ti}}{(\beta_{T11}J^2+\beta_{T12}J)^2}\partial\beta_{T12}\right]^2 + \left(-\frac{J}{\beta_{T11}J^2+\beta_{T12}J}\partial\beta_{Ti}\right)^2} \tag{36}$$

2.2.4 Compute the interaction between the propellers, in case of two propellers

2.2.4.1 Filter

In this case a different filter is needed compared to 2.1. The datapoint is filtered to select the tests that have sufficient and symmetric propeller rate and a small rudder angle, but all drift angles should be included. The propeller symmetry rate is checked according to 2.1.1. Tests of type STATX0, MULTI0 and MULTI1 are selected if the following conditions are fulfilled:

$$|n_1| > 0.3n_{MAX} \tag{37}$$

$$|\delta_1| < 5° \tag{38}$$

$$|\beta| > 1° \tag{39}$$

For the multi-modal tests only tests where the ship's accelerations are equal to zero are selected.

[1] For the second propeller it is assumed here that $K_{T2}(0) = K_{T1}(0)$

2.2.4.2 Regression setup and execution

The interaction between the two propellers is modelled with a drift dependent propeller rate acceleration factor:

$$T_{Pi} = \frac{\rho}{2}\left[\left[\left(1 - w_{Ti}(\varepsilon_{Ti}^{*})\right)u\right]^{2} + \left[0.7\pi n_{i}\left(1 + a_{Ti}(\beta)\right)D_{Pi}\right]^{2}\right]\frac{\pi D_{Pi}^{2}}{4}C_{T}(\varepsilon_{Ti}) \tag{40}$$

while implicitly

$$\varepsilon_{Ti}^{*} = \mathrm{atan2}\left(u, 0.7\pi n_{i}\left(1 + a_{Ti}(\beta)\right)D_{Pi}\right) \tag{41}$$

The corresponding ODRPack program is *PropellerIntT.exe*. The user may define the table of drift angles for which the acceleration rate should be computed. The acceleration factor is forced to zero when the drift angle is equal to 0° or ±180°.

The uncertainty of the acceleration rates is determined by ODRPack, however, the uncertainty of the wake factors is neglected (as is any uncertainty on the ship’s kinematic and control parameters).

2.3 Propeller shaft torque computation for each propeller i

2.3.1 Determine the slope of the linear KQ(J’) characteristic in the first quadrant

The same filter is used as in 2.2.1, but now the minimal and maximal propeller shaft torque coefficients need to be computed:

$$K_{Qi} = \frac{Q_{Pi}}{\rho n_{i}^{2} D_{Pi}^{5}} \tag{42}$$

The slope of the characteristic is computed using the same ODRPack program *KJacc.exe*. In this routine the slope β_{Qi} is determined:

$$K_{Qi}(J_{i}') = \beta_{Qi}J_{i}' + K_{Qi}(0) \tag{43}$$

$K_{Qi}(0)$ is the maximal propeller shaft torque coefficient measured at bollard pull conditions.

2.3.2 Update the open water characteristic of the propeller

The open water characteristic of the propeller is now corrected based on the measured propeller shaft torque during bollard pull condition.

The correction is performed per quadrant, under the assumption that the open water characteristic,

- follows a quadratic trend (Quadrants 1 and 3) between $J = 0$ and $J = J_{MAX}$;
- follows a 4th degree polynomial trend (Quadrants 2 and 4) between $J = -J_{MAX}$ and $J = 0$.

A filter groups the data of the open water characteristic per quadrant between the above described boundaries. Each quadrant uses the same regression program in ODRPack as for the thrust:

- Quadrant 1, *KJ1.exe*:

$$K_{Q}(J) = \beta_{Q11}J^{2} + \beta_{Q12}J + K_{Q1}(0) \tag{44}$$

  with $K_{Q1}(0)$ the maximal propeller shaft torque coefficient of propeller 1 measured at bollard pull conditions.

- Quadrant 2, *KJ2.exe*:

$$K_{Q}(J) = \beta_{Q21}J^{4} + \beta_{Q22}J^{3} + \beta_{Q23}J^{2} + \beta_{Q24}J + K_{Q1}(0) \tag{45}$$

  with $K_{Q}(0)$ the minimal propeller shaft torque coefficient measured at bollard pull conditions.

- Quadrant 3, *KJ3.exe*:

$$K_Q(J) = \beta_{Q31}J^2 + \beta_{Q32}J + K_{Q1}(0) \quad (46)$$

with $K_Q(0)$ the minimal propeller shaft torque coefficient measured at bollard pull conditions.

- Quadrant 4, *KJ4.exe*:

$$K_Q(J) = \beta_{Q41}J^4 + \beta_{Q42}J^3 + \beta_{Q43}J^2 + \beta_{Q44}J + K_{Q1}(0) \quad (47)$$

with $K_Q(0)$ the maximal propeller shaft torque coefficient measured at bollard pull conditions.

After the execution of these four programs a corrected open water characteristic $C_{Qcorr}(\varepsilon)$ is created as follows:

- Within the boundaries of the regression:

$$C_{Qcorr}(\varepsilon) = \frac{K_Q(J)}{\frac{\pi}{8}(J^2+(0.7\pi)^2)} \quad (48)$$

with $K_Q(J)$ equal to one of the above equations.
- Outside the boundaries of the regression, the original values of $C_Q(\varepsilon)$ are taken.

2.3.3 Compute the wake factors

The computation of the wake factors is completely analogous to 2.2.3, but now the propeller shaft torque identity is used

$$K_{Qi}(J_i') = \beta_{Qi}J_i' + K_{Qi}(0) \equiv K_Q(J) = \beta_{Q11}J^2 + \beta_{Q12}J + K_{Q1}(0) \quad (49)$$

$$J_i' = \frac{\beta_{Q11}J^2+\beta_{Q12}J}{\beta_{Qi}} \quad (50)$$

and

$$w_{Qi} = \frac{J_i'-J}{J_i'} = \frac{\beta_{Q11}J^2+(\beta_{Q12}-\beta_{Qi})J}{\beta_{Q11}J^2+\beta_{Q12}J} \quad (51)$$

2.3.4 Compute the interaction between the propellers, in case of two propellers

The same filter is used as in 2.2.4. The interaction between the two propellers is again modelled with a drift dependent propeller rate acceleration factor:

$$Q_{Pi} = \frac{\rho}{2}\left[\left[\left(1 - w_{Qi}(\varepsilon_{Qi}^*)\right)u\right]^2 + \left[0.7\pi n_i\left(1 + a_{Qi}(\beta)\right)D_{Pi}\right]^2\right]\frac{\pi D_{Pi}^2}{4}D_{Pi}C_Q(\varepsilon_{Qi}) \quad (52)$$

while implicitly

$$\varepsilon_{Qi}^* = \text{atan2}\left(u, 0.7\pi n_i\left(1 + a_{Qi}(\beta)\right)D_{Pi}\right) \quad (53)$$

The corresponding ODRPack program is *PropellerIntQ.exe*. The user may define the table of drift angles for which the acceleration rate should be computed. The acceleration factor is forced to zero when the drift angle is equal to 0° or ±180°.

2.4 Modelled thrust and torque

2.4.1 Symmetric interaction factors between the propellers, in case of two propellers

The interaction factors computed by the regression routine, e.g. $a_{Q1}(\beta)$ and $a_{Q2}(\beta)$, are made symmetric as follows:

- $a'_{Q1}(\beta) = \frac{1}{2}\left(a_{Q1}(\beta) + a_{Q2}(-\beta)\right)$;
- $a'_{Q2}(\beta) = a'_{Q1}(-\beta)$

These factors are saved and may be used as input for the manoeuvring simulator, but are not used to compute the result of the mathematical model.

2.4.2 Expression for thrust and torque

Using the determined regression coefficients, the mathematical model for the thrust and the torque can be computed:

$$T_{Pi} = \frac{0.7^2}{8}\pi^3 \rho n_{Ti}^2 D_{Pi}^4 (1 + \tan^2 \varepsilon_{Ti}) C_T(\varepsilon_{Ti}) \quad (54)$$

$$Q_{Pi} = \frac{0.7^2}{8}\pi^3 \rho n_{Qi}^2 D_{Pi}^5 (1 + \tan^2 \varepsilon_{Qi}) C_Q(\varepsilon_{Qi}) \quad (55)$$

Mind that the harmonic yaw tests were not included in the regression. The effect of the yaw rate is modelled with the so-called course angle ψ^*:

$$\psi^* = \beta + \gamma \ (|\beta| \leq 90°) \quad (56)$$

$$\psi^* = \beta + \gamma - 180° \ (|\beta| > 90°) \quad (57)$$

with the yaw angle γ

$$\gamma = \text{atan2}\left(r\frac{L_{PP}}{2}, u\right) \quad (58)$$

In case of a twin propeller setup, the propeller acceleration factor should be computed with the course angle.

In Regstatx the SOWStatistics library is used to check the agreement between the measured and modelled values. The modelled thrust and torque of each propeller are added to the datapoint.

3 Steering computation

Remark: a mathematical model for the thrust of the propellers need to be available.

3.1 Determine the neutral rudder angle for each rudder i

3.1.1 Filter data

Straight line tests have to be selected which have sufficient and symmetric propeller rate (see 2.1.1) and varying rudder angles. The filtering has to be performed in distinct groups j (one set per test name and per given propeller rate) for further regression analysis.

Tests of type MULTI0, MULTI1 and PAAL are selected if the following conditions are fulfilled:

$$|n_1| > 0.25 n_{MAX} \quad (59)$$

$$|\delta_1| \neq 0° \quad (60)$$

$$|\beta| < 2° \quad (61)$$

For the multi-modal tests only tests where the ship's accelerations are equal to zero are selected.

3.1.2 Regression setup and execution

The neutral rudder angle δ_{i0} is defined as the rudder angle at which the normal force acting on the rudder turns zero. The regression is carried out for each group j – provided there are more than 20 datapoint rows in the group – with the ODRPack program *d0.exe*:

$$\delta_{ij} = \beta_{i1j} F_N^3 + \beta_{i2j} F_N^2 + \beta_{i3j} F_N + \delta_{i0j} \quad (62)$$

The regression results from each group are then combined to:

- The neutral rudder angle:

$$\delta_{i0} = \frac{\sum_{j=1}^{m} \delta_{i0j}}{m} \quad (63)$$

- The neutral rudder angle's uncertainty:

$$\partial\delta_{i0} = \frac{\sqrt{\sum_{j=1}^{m} \partial\delta_{i0j}^2}}{m} \quad (64)$$

with m the number of distinct groups with sufficient rows.

If two rudders are considered, the results can be made symmetric:

- $\delta'_{10} = \frac{sign\ \delta_{10}}{2}(|\delta_{10}| + |\delta_{20}|)$
- $\delta'_{20} = \frac{sign\ \delta_{20}}{2}(|\delta_{10}| + |\delta_{20}|)$

These neutral angles are saved and may be used as input for the manoeuvring simulator, but are not used to compute the result of the mathematical model.

3.2 Determine the inflow to the rudder behind the propeller jet

3.2.1 Overview

The next step is to determine the inflow velocity u_{R0} to the rudder, including the propeller jet behind the hull, but without the additional effect of the hull on the rudder, which will be modelled by the regression analysis. For this purpose the mathematical model for the thrust of the propellers need to be available and all coefficients of that model need to be read. The magnitude of the jet, depends on the position of the rudder behind the propeller, defined by the parameter k (see Table 2), and the ratio between the propeller diameter and rudder height η.

Table 2 - Relationship between the parameter k and the distance between the rudder stock and the propeller tips x_{RS}.

x_{RS}/D_P	0.00	0.25	0.50	0.75	1.00
k	0.50	0.79	0.88	0.94	0.96

3.2.2 Determine the boundaries for the applicable u_{R0}-model in the fourth quadrant

Due to the opposite flows in the fourth quadrant the impulse theory predicts a bifurcation. The bifurcation angles are referred to as A and B, see (Delefortrie *et al.*, 2010):

$$A = 0 \Leftrightarrow \sin\varepsilon = -k\sqrt{\frac{|C_T(\varepsilon)|}{\left[1+2k+\sqrt{\frac{1-\eta}{\eta}}\right]\left[1+\sqrt{\frac{1-\eta}{\eta}}\right]}} \quad (65)$$

$$B = 0 \Leftrightarrow \sin\varepsilon = -k\sqrt{\frac{|C_T(\varepsilon)|}{\left[1+2k-\sqrt{\frac{1-\eta}{\eta}}\right]\left[1-\sqrt{\frac{1-\eta}{\eta}}\right]}} \quad (66)$$

These implicit equations are solved numerically. The functions are evaluated for all ε in the open water characteristic (4th quadrant). A sign change indicates the root values for A, B are found. If η < 2/3, it is possible that no roots will be found for B. In this case B is equal to -90°.

3.2.3 Compute the inflow velocity

Prior to the computation the propulsion model coefficients have to be computed to serve as input. The computation is different per quadrant:

- First quadrant:

$$u_{R0i} = \frac{1}{1-w_{Ti}}\sqrt{\frac{\left\{\eta\left[(1-k)\sin\varepsilon_{Ti} + k\sqrt{C_T+\sin^2\varepsilon_{Ti}}\right]^2 + (1-\eta)\sin^2\varepsilon_{Ti}\right\}}{\{[(1-w_{Ti})u]^2+[0.7\pi n_{Ti}D_{Pi}]^2\}}} \quad (67)$$

- Second and third quadrant: $u_{R0i} = u$
- In the fourth quadrant the situation is more complex:
    - for a propeller loading $\varepsilon_{Ti} \leq B$ the inflow velocity is determined by:

$$u_{R0i}(\varepsilon_{Ti}) = \frac{1}{1-w_{Ti}}sgn(u_{RP})\sqrt{\frac{\left\{\eta\left[(1+k)\sin\varepsilon_{Ti} + k\sqrt{|C_T|+\sin^2\varepsilon_{Ti}}\right]^2 - sgn(u_{RP})(1-\eta)\sin^2\varepsilon_{Ti}\right\}}{\{[(1-w_{Ti})u]^2+[0.7\pi n_{Ti}D_{Pi}]^2\}}} \quad (68)$$

    and $sgn(u_{RP})$ is negative;
    - for a propeller loading $\varepsilon_{Ti} \geq A$ the inflow velocity is determined by the average of:

$$u_{R0i}(\varepsilon_{Ti}) = \frac{1}{1-w_{Ti}} sgn(u_{RP}) \sqrt{\frac{\left\{\eta\left[(1+k)\sin\varepsilon_{Ti} + k\sqrt{|C_T| + \sin^2\varepsilon_{Ti}}\right]^2 - sgn(u_{RP})(1-\eta)\sin^2\varepsilon_{Ti}\right\}}{\{[(1-w_{Ti})u]^2 + [0.7\pi n_{Ti} D_{Pi}]^2\}}} \quad (69)$$

and

$$u_{R0i}(\varepsilon_{Ti}) = \frac{1}{1-w_{Ti}} sgn(u_{RP}) \sqrt{\frac{\left\{\eta\left[(1-k)\sin\varepsilon_{Ti} + k\sqrt{|C_T| + \sin^2\varepsilon_{Ti}}\right]^2 - sgn(u_{RP})(1-\eta)\sin^2\varepsilon_{Ti}\right\}}{\{[(1-w_{Ti})u]^2 + [0.7\pi n_{Ti} D_{Pi}]^2\}}} \quad (70)$$

with $sgn(u_{RP})$ positive;

- for intermediate points a linear interpolation, based on the propeller loading angle, is applied between the inflow velocity at angle A and the inflow velocity at angle B. To compute the above formulae at angle A (B) the following changes are needed:

$$\varepsilon_{Ti} \rightarrow A(B)$$

$$n_{Ti} \rightarrow \frac{u}{0.7\pi D_{Pi} \tan A(B)}$$

  The wake factor w_{Ti} is not adapted at A (B). The inflow velocity is then:

$$u_{R0i}(\varepsilon_{Ti}) = \frac{(A-\varepsilon_{Ti})u_{R0i}(B) + (\varepsilon_{Ti}-B)u_{R0i}(A)}{A-B} \quad (71)$$

In the above equations is assumed that a twin rudder ship has two propellers. This is not always the case, e.g. Myzako. For a ship equipped with two rudders and a single propeller, the inflow velocity to rudder 2 is assumed to be equal to the inflow velocity to rudder 1. In case of a twin propeller setup, the propeller acceleration factor should be computed with the course angle ψ^* instead of the drift angle.

The lateral inflow velocity to the rudder is always equal to:

$$v_R = v - r\frac{L_{PP}}{2} \quad (72)$$

The inflow velocities are added to the datapoint as new columns.

3.3 Compute the wake factors in the first quadrant

3.3.1 Filter data

Tests of type PAAL, STATX0, MULTI0 and MULT1 in the first quadrant with rudder and propeller action, but without ship accelerations, are filtered. The propeller rate is symmetric (see 2.1.1). The following conditions must be fulfilled:

$$|n_1| > 0.25 n_{MAX} \quad (73)$$

$$0 \leq u \leq 1 \text{ m/s} \quad (74)$$

$$|\delta| > 1° \quad (75)$$

The wake factors are computed as a function of the rudder angle, separately for a given set of drift angles. The user may define both the table of rudder and drift angles for which the wake factors rate should be computed. For every drift angle β_{table}, the appropriate tests are selected so that:

$$|\beta_{table} - \beta| < 1° \quad (76)$$

3.3.2 Determine the forces

For each rudder i, each drift angle and each force component the mathematical model is determined:

- Longitudinal force, ODRPack program *wrxl.exe*:

$$F_{Xi} = \frac{\rho}{2} A_R \left[\left(\left(1 - w_{RXi}(\delta)\right) u_{R0i} \right)^2 + v_R^2 \right] \left[C_{Li}(\alpha_{Ri}) \sin \beta_{Ri} + C_{Di}(\alpha_{Ri}) \cos \beta_{Ri} \right] \quad (77)$$

- Lateral force, ODRPack program *wryl.exe*:

$$F_{Yi} = \frac{\rho}{2} A_R \left[\left(\left(1 - w_{RYi}(\delta)\right) u_{R0i} \right)^2 + v_R^2 \right] \left[C_{Li}(\alpha_{Ri}) \cos \beta_{Ri} - C_{Di}(\alpha_{Ri}) \sin \beta_{Ri} \right] \quad (78)$$

- Steering torque, ODRPack program *wrql.exe*[2]:

$$Q_{Ri} = \frac{\rho}{2} A_R c \left[\left(\left(1 - w_{RQi}(\delta)\right) u_{R0i} \right)^2 + v_R^2 \right] C_{QRi}(\alpha_{Ri}) \quad (79)$$

For the computation of the wake factors, it is important to have initial values for the regression coefficients different from zero. In Regstatx4, the initial values are 0.5. For the computation of w_{RYi} and w_{RQi} boundary conditions are applied within the ODRPack sources:

$$0 \le w_R \le 1 \quad (80)$$

The rudder angle δ in the equations is equal to:

$$\delta = \delta_R - \delta_0 \quad (81)$$

δ_R being the measured rudder angle. The local drift angle near the rudder is:

$$\beta_R = \text{atan2}(-v_R, (1 - w_R) u_R) \quad (82)$$

and the total inflow angle:

$$\alpha_R = \beta_R + \delta \quad (83)$$

The lift, drag and torque coefficients are found through the execution of open water tests. The obtained wake factors and their standard deviations are saved per rudder, force component and drift angle. In some cases additional corrections are needed, for instance, when the range of measured rudder angles is smaller than the input table. In this case a constant wake factor is assumed for the outer boundaries, however for some ships additional corrections may be needed in the Regstatx source code.

Once all wake computations are finished, the wake factors are grouped per rudder and per rudder component in a 2D table (rudder angle, drift angle). Symmetric variants are computed as well (only for simulation purposes):

- For ships equipped with a single rudder:
  - $w_R'(\delta_R, \beta) = \frac{1}{2}\left(w_R(\delta_R, \beta) + w_R(-\delta_R, -\beta)\right);$
- For ships with two rudders:
  - $w_{R1}'(\delta_R, \beta) = \frac{1}{2}\left(w_{R1}(\delta_R, \beta) + w_{R2}(-\delta_R, -\beta)\right);$
  - $w_{R2}'(\delta_R, \beta) = w_{R1}'(-\delta_R, -\beta)$

[2] The results obtained by this model are not yet satisfactory.

3.4 Compute the propeller acceleration in the third quadrant

3.4.1 Filter

Tests of type PAAL, STATX0, MULTI0 and MULT1 in the third quadrant with rudder and propeller action, but without ship accelerations, are filtered. The propeller rate is symmetric (see 2.1.1). The following conditions must be fulfilled:

$$|n_1| < -0.25 n_{MAX} \tag{84}$$

$$u \leq 0 \text{ m/s} \tag{85}$$

$$v = 0 \text{ m/s} \tag{86}$$

$$|\delta| > 0° \tag{87}$$

3.4.2 Regression

The lateral rudder force in the third quadrant is determined with the ODRPack program *wrIII.exe*:

$$F_{Yi} = \frac{\rho}{2} A_R \left[u_{Ri}^2 + v_R^2\right]\left[C_{Li}(\alpha_{Ri}) \cos\beta_{Ri} - C_{Di}(\alpha_{Ri}) \sin\beta_{Ri}\right] \tag{88}$$

with

$$u_{Ri} = \beta_{3i} n_i + (1 - w_{RYi}) u_{R0i} \tag{89}$$

The obtained propeller acceleration factor β_{3i} is assumed to be applicable to the other rudder force components as well. For two rudders a symmetric factor is computed (only for simulation purposes):

$$\beta_3' = \frac{1}{2}(\beta_{31} + \beta_{32}) \tag{90}$$

3.5 Modelled forces acting on the rudder

It is important to point out that the wake factors obtained in the first quadrant are applicable to all quadrants. These wake factors are expressed in a 2D table, with drift angles limited to the first quadrant. Harmonic yaw tests were not included to compute the wake factors. The effect of the yaw rate is expressed by using ψ^* instead of the drift angle.

For lookup purposes, the course angle has to be limited in the 3rd and 4th quadrant:

$$\psi_{90}^* = 180 - \psi^* (\psi^* > 90°) \tag{91}$$

$$\psi_{90}^* = -180 - \psi^* (\psi^* < -90°) \tag{92}$$

In all cases the course angle should be limited to the boundaries of the drift angle in the 2D table. With known rudder angle and course angle, the corresponding wake factors can be retrieved for each rudder and force component and the effective inflow to the rudder can be computed:

- Quadrants 1 and 4:

$$u_R = (1 - w_R) u_{R0} \tag{93}$$

- Quadrants 2 and 3:

$$u_R = \beta_3 n + (1 - w_R) u \tag{94}$$

The forces acting on the rudder are then:

$$F_X = \frac{\rho}{2} A_R [u_R^2 + v_R^2][C_L(\alpha_R) \sin\beta_R + C_D(\alpha_R) \cos\beta_R] \tag{95}$$

$$F_Y = \frac{\rho}{2} A_R [u_R^2 + v_R^2][C_L(\alpha_R) \cos\beta_R - C_D(\alpha_R) \sin\beta_R] \tag{96}$$

$$Q_R = \frac{\rho}{2} A_R c [u_R^2 + v_R^2] C_{QR}(\alpha_R) \tag{97}$$

In Regstatx the SOWStatistics library is used to check the agreement between the measured and modelled values. The modelled forces of each rudder are added to the datapoint.

4 Hull computation

4.1 Drift forces

4.1.1 Filter

The drift induced forces are computed for each force component (6 DOF). Each degree of freedom uses its own filter, but in general tests are selected in which the yaw rate is equal to zero and in which the rudder angles are limited. Asymmetric conditions are always excluded (see 2.1.1).

General filter criteria, applicable to all degrees of freedom are:

$$u \neq 0 \; or \; v \neq 0 \tag{98}$$

$$r = 0 \tag{99}$$

$$\dot{r} = 0 \tag{100}$$

$$|\delta| < 10° \text{ (if the steered propeller rate equals zero)} \tag{101}$$

$$|\delta| < 5° \; and \; |\delta_{steered}| < 3° \text{ (if the steered propeller rate differs from zero)} \tag{102}$$

The sub criteria depend on the degree of freedom:

- Longitudinal force (exclude tests of type PMMY2):
$$\dot{v} = 0 \tag{103}$$
- Lateral force, roll and yaw moment: additional criteria applicable when $n \neq 0$:
No tests of type PMMY2 or STATX0
$$\dot{v} = 0 \tag{104}$$
$$|\delta| = 0 \tag{105}$$
- Heave force and pitch moment: additional criteria applicable when $n \neq 0$:
No tests of type PMMY2 or STATX0
$$\dot{v} = 0 \tag{106}$$
- Roll and pitch moment:
$$|n| \leq 0.125 n_{MAX} \tag{107}$$

4.1.2 Regressions

4.1.2.1 Overview

Regressions are carried out with a separate program and input for each degree of freedom. Some of the programs use the thrust or the rudder forces as input:

- Preference is given to the modelled values, but if not available the measured values will be used;
- The standard models assume the presence of two propellers and two rudders, if these are not available, the corresponding forces are equal to zero.

Regressions are always carried out for the 6 degrees of freedom and compute the drift dependency for a table of drift angles. The user has the opportunity to change the values of the input table. Moreover, it is possible to use drift functions computed with other means and have them read by Regstatx.

4.1.2.2 Longitudinal force

The corresponding ODRPack program is *Xbeta.exe*:

$$X = (X'_{\dot{u}} - 1)m\dot{u} + \frac{\rho}{2}LT(u^2 + v^2)X'(\beta) + \sum F_{Xi} + \left(1 - t(\beta)\right)\sum T_{Pi} \qquad (108)$$

The added inertia in the above and following formulae are always non dimensional, in agreement with the program code and documentation of ODRPack 5. The regression output of interest is $X'_{\dot{u}}$ and $X'(\beta)$. $X'_{\dot{u}}$ has two values: one for $u \geq 0$ and one for $u < 0$. The thrust deduction $t(\beta)$ is a preliminary model, which has no further use. A symmetric variant of the drift function is saved as well:

- $X''(\beta) = \frac{1}{2}\left(X'(\beta) + X'(-\beta)\right)$

4.1.2.3 Lateral force

The corresponding ODRPack program is *Ybeta.exe*:

$$Y = (Y'_{\dot{v}} - 1)m\dot{v} + \frac{\rho}{2}LT(u^2 + v^2)Y'(\beta) + \left(1 + a_H(\beta)\right)\sum F_{Yi} + Y_{PT}(\beta)\sum T_{Pi} \qquad (109)$$

The regression output of interest is $Y'_{\dot{v}}$ and $Y'(\beta)$. $Y_{\dot{v}}$ has two values: one for $u \geq 0$ and one for $u < 0$. The functions $a_H(\beta)$ and $Y_{PT}(\beta)$ are preliminary models, without further use. A symmetric variant of the drift function is saved as well:

- $Y''(\beta) = \frac{1}{2}\left(Y'(\beta) - Y'(-\beta)\right)$;
- $Y''(\beta = 0) = 0$

4.1.2.4 Heave force

The corresponding ODRPack program is *Zbeta.exe*:

$$Z = (Z'_{\dot{v}}|\dot{v}| + Z'_{\dot{u}}\dot{u})m + mgT_{rh}^2 Z'(\beta) + T_{rh}Z_{PT}(\beta)\sum T_{Pi} \qquad (110)$$

The regression output of interest are the retardation terms $Z'_{\dot{v}}$ and $Z'_{\dot{u}}$ and the drift function $Z'(\beta)$. The function $Z_{PT}(\beta)$ is a preliminary model, without further use. A symmetric variant of the drift function is saved as well:

- $Z''(\beta) = \frac{1}{2}\left(Z'(\beta) + Z'(-\beta)\right)$

4.1.2.5 Roll moment

The corresponding ODRPack program is *Kbeta.exe*:

$$K = (K'_{\dot{v}}T + z_G)m\dot{v} + \rho LT^2(u^2 + v^2)K'(\beta) \qquad (111)$$

The regression output of interest is $K'_{\dot{v}}$ and $K'(\beta)$. A symmetric variant of the drift function is saved as well:

- $K''(\beta) = \frac{1}{2}\left(K'(\beta) - K'(-\beta)\right)$;
- $K''(\beta = 0) = 0$

The regression coefficients are made dimensionless with the ship's draft, but this needs to be investigated more thoroughly.

4.1.2.6 Pitch moment

The corresponding ODRPack program is *Mbeta.exe*:

$$M = [M'_{\dot{v}} L|\dot{v}| + (M'_{\dot{u}} L - z_G)\dot{u}]m + mgLT_{rh}^2 M'(\beta) \quad (112)$$

The regression output of interest are $M'_{\dot{v}}$ (retardation term), $M'_{\dot{u}}$ and the drift function $M'(\beta)$. A symmetric variant of the drift function is saved as well:

- $M''(\beta) = \frac{1}{2}\left(M'(\beta) + M'(-\beta)\right)$

4.1.2.7 Yaw moment

The corresponding ODRPack program is *Nbeta.exe*:

$$N = (N'_{\dot{v}} L - x_G)m\dot{v} + (N'_{\dot{u}} L + y_G)m\dot{u} + \frac{\rho}{2} L^2 T(u^2 + v^2)N'(\beta) +$$

$$\left(x_R + a_H x_H(\beta)\right) \sum F_{Yi} + N_{PT}(\beta) \sum T_{Pi} - \sum y_i (F_{Xi} + T_{Pi}) \quad (113)$$

The regression output of interest are $N'_{\dot{u}}$ (0 if $y_G = 0$), $N'_{\dot{v}}$ and $N'(\beta)$. $N_{\dot{v}}$ has two values: one for $u \geq 0$ and one for $u < 0$. The functions $a_H x_H(\beta)$ and $N_{PT}(\beta)$ are preliminary models, without further use. The term $\sum y_i (F_{Xi} + T_{Pi})$ considers the additional induced moment in a twin rudder, twin propeller setup, where y_i is the lateral position of propeller/rudder i.

A symmetric variant of the drift function is saved as well:

- $N''(\beta) = \frac{1}{2}\left(N'(\beta) - N'(-\beta)\right)$;
- $N''(\beta = 0) = 0$

4.1.3 Result

After the computation of the drift forces and the surge and sway added inertia, the model results are computed and added as new hull model columns to the datapoint.

4.2 Yaw and yaw-drift correlation forces

4.2.1 Filter

The yaw and yaw-drift correlation forces are computed simultaneously for each force component (6 DOF). The used filter is the same for all DOF. Asymmetric conditions are always excluded (see 2.1.1). Tests of type PMMPSI2 or OSCPSI are selected with the following criteria:

$$n_{steered} = 0 \quad (114)$$

$$|n| < 50\ rpm \quad (115)$$

$$|\delta_{steered}| < 3° \quad (116)$$

$$|\delta| < 5° \quad (117)$$

4.2.2 Regressions

4.2.2.1 Overview

Regressions are carried out with a separate program and input for each degree of freedom. Some of the programs use the rudder forces as input:

- Preference is given to the modelled values, but if not available the measured values will be used;
- The standard models assume the presence of two rudders, if these are not available, the corresponding forces are equal to zero.

Regressions are always carried out for the 6 degrees of freedom and compute the yaw γ and yaw-drift χ dependency for a table of yaw and yaw-drift correlation angles. In some cases the yaw acceleration derivative is also expressed as a function of the drift angle. The user has the opportunity to change the values of the input table. Moreover, it is possible to use yaw and yaw-drift correlation functions computed with other means and have them read by Regstatx.

The dependent variable of the regression is the measured force component minus the drift dependent part modelled in 4.1.2. As a consequence some boundary conditions are needed for the function values:

$$F'(\gamma = 0°) = F'(\gamma = \pm 180°) = 0 = F'(\chi = \pm 180°) = F'(\chi = \pm 90°) = F'(\chi = 0°) \qquad (118)$$

4.2.2.2 Longitudinal force

The corresponding ODRPack program is *Xgammachi.exe*:

$$X = mvr + mx_G r^2 + (X'_{\dot{r}} L_{PP} + y_G) m\dot{r}$$
$$+\frac{\rho}{2} LT\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) X'(\gamma) + \frac{\rho}{2} LT\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) X'(\chi) + \sum F_{Xi} \qquad (119)$$

The regression output of interest is $X'_{\dot{r}}$ (0 if $y_G = 0$), $X'(\gamma)$ and $X'(\chi)$. The functions $X'(\gamma)$ and $X'(\chi)$ have different values depending on the longitudinal speed: a set for $u \geq 0$ and a set for $u < 0$. Symmetric variants of these functions are saved as well:

- $X''(\gamma) = \frac{1}{2}\left(X'(\gamma) + X'(-\gamma)\right)$;
- $X''(\chi) = \frac{1}{2}\left(X'(\chi) + X'(\chi + 180°)\right)$

4.2.2.3 Lateral force

The corresponding ODRPack program is *Ygammachi.exe*:

$$Y = -mur + my_G r^2 + (Y'_{\dot{r}}(\beta) L_{PP} - x_G) m\dot{r}$$
$$+\frac{\rho}{2} LT\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) Y'(\gamma) + \frac{\rho}{2} LT\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) Y'(\chi) + \sum F_{Yi} \qquad (120)$$

The regression output of interest is $Y'_{\dot{r}}(\beta)$, $Y'(\gamma)$ and $Y'(\chi)$. The functions $Y'(\gamma)$ and $Y'(\chi)$ have different values depending on the longitudinal speed: a set for $u \geq 0$ and a set for $u < 0$. Symmetric variants of all functions are saved as well:

- $Y''_{\dot{r}}(\beta) = \frac{1}{2}\left(Y'_{\dot{r}}(\beta) + Y'_{\dot{r}}(-\beta)\right)$;
- $Y''(\gamma) = \frac{1}{2}\left(Y'(\gamma) - Y'(-\gamma)\right)$;
- $Y''(\chi) = \frac{1}{2}\left(Y'(\chi) - Y'(\chi + 180°)\right)$

4.2.2.4 Heave force

The corresponding ODRPack program is *Zgammachi.exe*:

$$Z = Z'_{\dot{r}} L_{PP} m|\dot{r}| + \frac{\rho}{2} LT\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) Z'(\gamma) + \frac{\rho}{2} LT\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) Z'(\chi) \tag{121}$$

The regression output of interest are the retardation term $Z'_{\dot{r}}$ and the functions $Z'(\gamma)$ and $Z'(\chi)$, which have different values depending on the longitudinal speed: a set for $u \geq 0$ and a set for $u < 0$. Symmetric variants of the functions are saved as well:

- $Z''(\gamma) = \frac{1}{2}\left(Z'(\gamma) + Z'(-\gamma)\right)$;
- $Z''(\chi) = \frac{1}{2}\left(Z'(\chi) + Z'(\chi + 180°)\right)$

4.2.2.5 Roll moment

The corresponding ODRPack program is *Kgammachi.exe*:

$$K = mz_G ur + K'_{\dot{r}}(\beta) T^2 m\dot{r} + \frac{\rho}{2} LT^2\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) K'(\gamma) + \frac{\rho}{2} LT^2\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) K'(\chi) \tag{122}$$

The regression output of interest is $K'_{\dot{r}}(\beta)$, $K'(\gamma)$ and $K'(\chi)$. The functions $K'(\gamma)$ and $K'(\chi)$ have different values depending on the longitudinal speed: a set for $u \geq 0$ and a set for $u < 0$. Symmetric variants of all functions are saved as well:

- $K''_{\dot{r}}(\beta) = \frac{1}{2}\left(K'_{\dot{r}}(\beta) + K'_{\dot{r}}(-\beta)\right)$;
- $K''(\gamma) = \frac{1}{2}\left(K'(\gamma) - K'(-\gamma)\right)$;
- $K''(\chi) = \frac{1}{2}\left(K'(\chi) - K'(\chi + 180°)\right)$

4.2.2.6 Pitch moment

The corresponding ODRPack program is *Mgammachi.exe*:

$$M = M'_{\dot{r}} L^2 m|\dot{r}| + \frac{\rho}{2} L^2 T\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) M'(\gamma) + \frac{\rho}{2} L^2 T\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) M'(\chi) \tag{123}$$

The regression output of interest are the retardation term $M'_{\dot{r}}$ and the functions $M'(\gamma)$ and $M'(\chi)$, which have different values depending on the longitudinal speed: a set for $u \geq 0$ and a set for $u < 0$. Symmetric variants of the functions are saved as well:

- $M''(\gamma) = \frac{1}{2}\left(M'(\gamma) + M'(-\gamma)\right)$;
- $M''(\chi) = \frac{1}{2}\left(M'(\chi) + M'(\chi + 180°)\right)$

4.2.2.7 Yaw moment

The corresponding ODRPack program is *Ngammachi.exe*:

$$\begin{aligned} N = & -mx_G ur - my_G vr + \left(N'_{\dot{r}}(\beta)\, mL^2 - I_{ZZ}\right)\dot{r} + x_R \sum F_{Yi} - \sum y_{Ri} F_{Xi} \\ & + \frac{\rho}{2} L^2 T\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) N'(\gamma) + \frac{\rho}{2} L^2 T\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right) N'(\chi) \end{aligned} \tag{124}$$

The regression output of interest is $N'_{\dot{r}}(\beta)$, $N'(\gamma)$ and $N'(\chi)$. The functions $N'(\gamma)$ and $N'(\chi)$ have different values depending on the longitudinal speed: a set for $u \geq 0$ and a set for $u < 0$. Symmetric variants of all functions are saved as well:

- $N''_{\dot{r}}(\beta) = \frac{1}{2}\left(N'_{\dot{r}}(\beta) + N'_{\dot{r}}(-\beta)\right)$;
- $N''(\gamma) = \frac{1}{2}\left(N'(\gamma) - N'(-\gamma)\right)$;
- $N''(\chi) = \frac{1}{2}\left(N'(\chi) - N'(\chi + 180°)\right)$

4.2.3 Result

After the computation of the yaw forces, yaw-drift correlation forces and the yaw added inertia, the model results are computed and the hull model columns of the datapoint are updated. The final model formulations are as follows:

$$X_H = \frac{\rho}{2}LT(u^2 + v^2)X'(\beta) + \frac{\rho}{2}LT\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)X'(\gamma) + \frac{\rho}{2}LT\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)X'(\chi) \quad (125)$$

$$X_{AI} = [X'_{\dot{u}}\dot{u} + X'_{\dot{r}}\, L\dot{r}]m \quad (126)$$

$$Y_H = \frac{\rho}{2}LT(u^2 + v^2)Y'(\beta) + \frac{\rho}{2}LT\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)Y'(\gamma) + \frac{\rho}{2}LT\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)Y'(\chi) \quad (127)$$

$$Y_{AI} = [Y'_{\dot{v}}\dot{v} + Y'_{\dot{r}}(\beta)L\dot{r}]m \quad (128)$$

$$Z_H = mgT_{rh}^2 Z'(\beta) + \frac{\rho}{2}LT\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)Z'(\gamma) + \frac{\rho}{2}LT\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)Z'(\chi) \quad (129)$$

$$Z_{AI} = [Z'_{\dot{u}}\,\dot{u} + Z'_{\dot{v}}\,|\dot{v}| + Z'_{\dot{r}}L|\dot{r}|]m \quad (130)$$

$$K_H = \rho LT^2(u^2 + v^2)K'(\beta) + \frac{\rho}{2}LT^2\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)K'(\gamma) + \frac{\rho}{2}LT^2\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)K'(\chi) \quad (131)$$

$$K_{AI} = [K'_{\dot{v}}\,\dot{v} + K'_{\dot{r}}(\beta)T\dot{r}]mT \quad (132)$$

$$M_H = mgLT_{rh}^2 M'(\beta) + \frac{\rho}{2}L^2T\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)M'(\gamma) + \frac{\rho}{2}L^2T\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)M'(\chi) \quad (133)$$

$$M_{AI} = [M'_{\dot{u}}\dot{u} + M'_{\dot{v}}|\dot{v}| + M'_{\dot{r}}L|\dot{r}|]mL \quad (134)$$

$$N_H = \frac{\rho}{2}L^2T(u^2 + v^2)N'(\beta) + \frac{\rho}{2}L^2T\left(u^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)N'(\gamma) + \frac{\rho}{2}L^2T\left(v^2 + \left(\frac{rL_{PP}}{2}\right)^2\right)N'(\chi) \quad (135)$$

$$N_{AI} = [N'_{\dot{u}}\dot{u} + N'_{\dot{v}}\dot{v} + N'_{\dot{r}}(\beta)\, L\dot{r}]mL \quad (136)$$

5 Propulsion induced computation

Remark: mathematical models for the thrust of the propellers and for the hull forces need to be available.

5.1 Compute thrust deduction

5.1.1 Filter data

The aim is to filter the tests that have sufficient propulsion, but with a limited rudder angle. Propulsion and steering do not have to be symmetric. The general criteria are:

$$|\delta_1| < 10° \tag{137}$$

$$|\delta_2| < 10° \tag{138}$$

Additional criteria are applicable depending on the test type:

- Type STATX0:

$$\delta_{steered1} = 0° \tag{139}$$
$$\delta_{steered2} = 0° \tag{140}$$
$$|n_1| > 0.3n_{MAX} \text{ or } |n_2| > 0.3n_{MAX} \tag{141}$$

- Type MULTI0/MULTI1:

$$|\delta_{steered1}| < 5° \tag{142}$$
$$|\delta_{steered2}| < 5° \tag{143}$$
$$|n_1| > 0.3n_{MAX} \text{ or } |n_2| > 0.3n_{MAX} \tag{144}$$
$$\dot{u} = \dot{v} = \dot{r} = 0 \tag{145}$$

- Type PAAL:

$$|\delta_{steered1}| < 5° \tag{146}$$
$$|\delta_{steered2}| < 5° \tag{147}$$

- Type PMMPSI2:

$$\delta_{steered1} = 0° \tag{148}$$
$$\delta_{steered2} = 0° \tag{149}$$
$$n_1 > 0.3n_{MAX} \text{ or } n_2 > 0.3n_{MAX} \tag{150}$$
$$u < 0 \tag{151}$$

5.1.2 Perform regression

The dependent variable is the measured longitudinal force minus the modelled hull components (see section 4). The computation is performed with the ODRPack program *ThrustDeduction.exe*. The problem formulation is:

$$X_{RP} = \sum\left(1 - t_i(\varepsilon^*, \varphi^*, \gamma^*)\right)T_{Pi} + \sum F_{Xi} \tag{152}$$

with the thrust deduction factor expressed as:

$$t_i = f(\varepsilon_i^*) + q_1(\varepsilon_i^*)\beta_1\varphi_i^* + q_3(\varepsilon_i^*)\beta_2\varphi_i^* + q_4(\varepsilon_i^*)\beta_3\gamma_i^* \tag{153}$$

In the above equation:

$$\varphi_i^* = \text{atan2}(|v|, 0.7\pi n_i D_{Pi}) \tag{154}$$

$$\gamma_i^* = \text{atan2}\left(\left|\frac{0.5r}{L_{PP}}\right|, 0.7\pi n_i D_{Pi}\right) \tag{155}$$

The quadrant identifiers q_1, q_3 and q_4 are equal to one when in the corresponding quadrant, else zero. The thrust deduction factors are tabulated for a set of ε^* angles which can be changed by the user. After the regression the thrust deduction factors are written in a 2D table $t(\varepsilon^*, \varphi^*)$:

$$-1 \leq t = f(\varepsilon^*) + q_1(\varepsilon^*)\beta_1\varphi^* + q_3(\varepsilon^*)\beta_2\varphi^* \leq 1 \qquad (156)$$

with uncertainty

$$\partial t = \sqrt{\left(\partial f(\varepsilon^*)\right)^2 + (q_1(\varepsilon^*)\partial\beta_1\varphi^*)^2 + (q_3(\varepsilon^*)\partial\beta_2\varphi^*)^2} \qquad (157)$$

The set of φ^* angles can also be changed by the user. The fourth quadrant correction $q_4(\varepsilon_i^*)\beta_3\gamma_i^*$ is written in a separate file.

5.1.3 Result

A column is added to the datapoint with the modelled propulsion induced longitudinal force:

$$X_P = \sum\left(1 - t_i(\varepsilon^*, \varphi^*, \gamma^*)\right)T_{Pi} \qquad (158)$$

Each thrust deduction factor is limited to:

$$-1 \leq t_i \leq 1 \qquad (159)$$

and is also added to the datapoint.

5.2 Compute oscillation characteristics

5.2.1 Overview

The aim is to determine the average value, amplitude, frequency and phase shift of oscillatory measurements of each degree of freedom, except for the longitudinal force. Each of the above components is added to the datapoint for each degree of freedom.

This is quite a long computation and is only carried out the first time a datapoint is read. To repeat the computation the original datapoint file from the towing tank must be used.

5.2.2 Filter

Oscillations occur in the second and in the fourth quadrant. Only tests of type STATX0 contain sufficient information to determine the oscillation characteristics:

- Second quadrant:

$$|n_1| < 0 \text{ or } |n_2| < 0 \text{ and } u > 0 \qquad (160)$$

- Fourth quadrant:

$$|n_1| > 0 \text{ or } |n_2| > 0 \text{ and } u < 0 \qquad (161)$$

As a result a number of test names is retrieved, corresponding to KRT-files which have to be read. In most cases STATX0 tests are carried out with different subtrajectories, with different rudder angles or propeller rates, and will have multiple references in the datapoint. In this case the KRT-file will be read multiple times, but each time only the lines corresponding to the current subtrajectory

$$n_{datapojnt\ i} - 20\ rpm \leq n_{KRT\ i} \leq n_{datapojnt\ i} + 20\ rpm \qquad (162)$$
$$\delta_{datapojnt\ i} - 4° \leq \delta_{KRT\ i} \leq \delta_{datapojnt\ i} + 4° \qquad (163)$$

will be taken into consideration. In this way a number of KRT (sub) files will be selected. With each sub file the following section will be carried out.

5.2.3 KRT regression

The aim is to perform a regression analysis on a single (part of) KRT-file for a given DOF. In each KRT file time series are given with a time step dt.

5.2.3.1 Pre processing

For each of the degrees of freedom prior computations are needed to obtain the value of the force or the moment:

- Sway force:

$$Y = Y_F + Y_A \tag{164}$$

- Heave force: first the mean sinkage has to be determined:

$$z = \frac{z_F|x_{z_F}| + z_A|x_{z_A}|}{|x_{z_F}| + |x_{z_A}|} \tag{165}$$

  If the sinkage is measured at four points, the average between the starboard and portside is chosen. Then the heave force can be computed using equation (15).

- Roll moment is known
- Pitch moment: first the trim has to be determined:

$$trim = \frac{z_A - z_F}{|x_{z_F}| + |x_{z_A}|} \tag{166}$$

  If the sinkage is measured at four points, the average between the starboard and portside is chosen. Then the pitch moment can be computed using equation (16).

- Yaw moment:

$$N = Y_F x_{Y_F} + Y_A x_{Y_A} \tag{167}$$

5.2.3.2 Determine initial values

Based on a Fast Fourier Transform analysis the optimal starting values for the regression in ODRPack are determined. The FFT is performed using the MathNet.Numerics dll (Fourier, BluesteinForward). The input consists of n time signals t which are transformed to n complex numbers $c = a + bi$ in the frequency domain:

- The average value is the average of the component at zero frequency: $\frac{a[0]}{n}$;
- The amplitude is the standard deviation of the time series t (multiplied with $\sqrt{2}$);
- The frequency is determined by the complex number with the largest magnitude at position i_{MAX}:

$$\omega = \frac{2\pi}{ndt} i_{MAX} \tag{168}$$

- The phase shift is also determined by the complex number with the largest magnitude at position i_{MAX}:

$$\varphi_* = -\text{atan2}(b[i_{MAX}], a[i_{MAX}]) \tag{169}$$

  The minus sign is needed, because the FFT is sine based, but ODRPack uses cosine.

5.2.3.3 Regression

The regression is carried out by the ODRPack program *Oscillations.exe*:

$$F = \beta_1 + \beta_2 \cos[\beta_3(t - t_0) + \beta_4] \tag{170}$$

Normally the coefficients β_1, β_3 and β_4 are saved as average, pulsation and phase shift. For the amplitude preference is given to the initial value (from the FFT). In case:

$$\beta_3 < 0.0198 \tag{171}$$

the initial values from the FFT are used for all harmonic components. Each of the above coefficients is added to the datapoint for each degree of freedom.

5.3 Compute oscillation model

5.3.1 Frequency model

5.3.1.1 Overview

The next step is to find for each DOF a relationship between the different harmonic components. In the first place the oscillation frequencies are considered, which are expressed with a quadratic model as a function of the apparent advance J'. The input frequencies are normally those of the yawing moment, but any other variable could be used. In general Y, K, M, N have similar frequencies. Z tends to have smaller frequencies (50%).

5.3.1.2 Filter

The filter from section 5.2.2 is narrowed as follows:

- Second quadrant:

$$|n_1| < 0.3n_{MAX} \text{ or } |n_2| < 0.3n_{MAX} \text{ and } u > 0 \tag{172}$$

- Fourth quadrant:

$$|n_1| > 0.3n_{MAX} \text{ or } |n_2| > 0.3n_{MAX} \text{ and } u < 0 \tag{173}$$

5.3.1.3 Regression

The used program in ODRPack is *KJ1.exe*, see 2.2.2, but in this case the constant term $K_T(0)$ is equal to zero and the dependent term is not $K_T(J)$, but the dimensionless frequency:

$$\omega' = \omega\sqrt{\frac{L}{g}} \tag{174}$$

The independent term is the apparent advance. In case of a twin propeller ship the maximal apparent advance of both propellers is used. The governing equation is then:

$$\omega'(J') = \beta_{T11}J'^2 + \beta_{T12}J' \tag{175}$$

As stated before, the input frequency is determined based on the yawing moment.

The outcome of the regression is expressed as a 2D table $\omega'(\varepsilon^*, \beta)$. The user can select the points for this table, however, the frequency is only depending of ε^*,

$$\omega'(\varepsilon^*) = \beta_{T11}(0.7\pi\tan\varepsilon_i^*)^2 + \beta_{T12}0.7\pi\tan\varepsilon_i^* \tag{176}$$

and the same values are copied for different β.

5.3.2 Average, amplitude and phase shift in the second quadrant

5.3.2.1 Filter

Asymmetric propulsion is acceptable, as long as both propellers operate in the same quadrant, and the maximal ε^* angle of both propellers is used. The rudder angles are restricted to:

$$|\delta_1| < 5° \tag{177}$$

$$|\delta_2| < 5° \tag{178}$$

whereas the propeller loading is limited by:

$$\varepsilon^* > \varepsilon_0^* \tag{179}$$

with ε_0^* the smallest propeller loading angle in the input table for the regressions. This table can be adapted by the user. Only tests of type STATX0 are selected or tests where all velocities and accelerations are equal to zero (bollard pull at negative propeller rate). In the latter case, the average component is the measured force, while the amplitude and phase shift of the oscillations are equal to zero.

5.3.2.2 Average and amplitude

The average force and the amplitude of the oscillations are determined as a function of the apparent propeller loading angle with the ODRPack program *OscillationForce.exe*:

$$F_0 = F_H + \mu \sum F_{PT}(\varepsilon_i^*) T_{Pi} \tag{180}$$

The hull component F_H is only applicable when the average force is considered and is zero when the amplitude is considered. μ represents the correct dimension:

- 1 for sway and heave;
- L for pitch and yaw;
- T for roll.

The dependent variable F_0 is the average term or the amplitude of the oscillations. In case of the average term, twin propeller action and yawing moment $\sum y_i X_{Pi}$ has to be subtracted. In case of the amplitude, a boundary condition is added that F_0 turns zero at bollard pull.

5.3.2.3 Phase shift

The phase shift is determined with the ODRPack program *OscillationPhase.exe*:

$$\varphi_* = \varphi_*(\varepsilon^*) \tag{181}$$

In case of twin propulsion the maximal ε^* angle of both propellers is used. At the smallest input angle, the phase shift is forced to zero. Values are always forced between $-\pi$ rad and π rad.

5.3.3 Average, amplitude and phase shift in the fourth quadrant

The filter is similar to the second quadrant. Asymmetric propulsion is acceptable, as long as both propellers operate in the same quadrant, and the maximal ε^* angle of both propellers is used. The rudder angles are restricted to:

$$|\delta_1| < 5° \tag{182}$$

$$|\delta_2| < 5° \tag{183}$$

whereas the propeller loading is limited by:

$$0 > \varepsilon^* > \varepsilon_0^* \tag{184}$$

with ε_0^* the smallest propeller loading angle in the input table for the regressions. This table can be adapted by the user. Only tests of type STATX0 are selected or tests where all velocities and accelerations are equal to zero (bollard pull at positive propeller rate). In the latter case, the average component is the measured force, while the amplitude and phase shift of the oscillations are equal to zero.

The computations of the average term, oscillation amplitude and phase shift is like in the second quadrant.

6 Steering induced computation

Remark: mathematical models for the rudder forces and for the propeller induced forces need to be available.

6.1 Quadrant 1

6.1.1 Filter

Tests in the first quadrant are selected:

$$u \geq 0 \tag{185}$$

$$n_1 \geq 0 \tag{186}$$

$$n_{steered\ 1} \geq 0 \tag{187}$$

In case of test types MULTI0 and MULTI1, only tests with accelerations equal to zero are selected. Asymmetric conditions are always excluded (see 2.1.1).

6.1.2 Regressions

6.1.2.1 Overview

Regression analysis is performed for each DOF, except the longitudinal force. In general the models are tabular in the apparent propeller loading angle ε^* and/or the course angle ψ^*. The input tables can be adapted by the user. The dependent variable is the measured force minus the modelled hull component and centrifugal terms.

6.1.2.2 Sway force

The regression is performed with the ODRPack program *YRP1.exe*:

$$Y_{RP} = (1 + a_H)\sum F_{Yi} + Y_{PT}(\psi^*)\sqrt{\frac{V^2}{gL}}\sum T_{Pi} + \frac{n}{n_{MAX}}[Y_{\dot{v}}^n \dot{v}m + Y_{\dot{r}}^n \dot{r}mL] \tag{188}$$

with

$$a_H = f_1(\psi^*)(\varepsilon^*)^2 + f_2(\psi^*)\varepsilon^* \tag{189}$$

After regression, the parameter a_H is transformed to a 2D table, with following limits:

$$0 \leq a_H(\varepsilon^*, \psi^*) \leq 3 \tag{190}$$

with corresponding standard deviation

$$\partial a_H(\varepsilon^*, \psi^*) = \sqrt{[\partial f_1(\psi^*)(\varepsilon^*)^2]^2 + [\partial f_2(\psi^*)\varepsilon^*]^2} \tag{191}$$

a_H is put equal to zero if for a certain course angle:

$$\partial f_1(\psi^*) > 100 \tag{192}$$

$$\partial f_2(\psi^*) > 100 \tag{193}$$

6.1.2.3 Heave force

Regression analysis is performed with the ODRPack program *ZRP1.exe*:

$$Z_{RP} = Z_{PT}(\psi^*)T_{rh}\sum T_{Pi} \tag{194}$$

6.1.2.4 Roll moment

Regression analysis is performed with the ODRPack program *KRP1.exe*:

$$K_{RP} = -(z_R + a_H z_H(\psi^*)T)\sum F_{Yi} + K_{PT}(\psi^*)T\sum T_{Pi} + \frac{n}{n_{MAX}}[K_{\dot{v}}^n \dot{v}mT + K_{\dot{r}}^n \dot{r}mT^2] \tag{195}$$

The values for a_H follow from a 2D interpolation of the model (see 6.1.2.2). If for a certain course angle ψ^{**}, $a_H(\varepsilon^*, \psi^{**})$ equals zero for all loading angles, a boundary condition is added, so that $z_H(\psi^{**})$ is zero as well.

6.1.2.5 Pitch moment

Regression analysis is performed with the ODRPack program *MRP1.exe*:

$$M_{RP} = z_{Hx}(\psi^*)L\sum F_{Xi} + M_{PT}(\psi^*)L\sum T_{Pi} \tag{196}$$

6.1.2.6 Yaw moment

Regression analysis is performed with the ODRPack program *NRP1.exe*:

$$N_{RP} = (x_R + a_H x_H(\psi^*)L)\sum F_{Yi} + N_{PT}(\psi^*)L\sum T_{Pi} + \frac{n}{n_{MAX}}[N_{\dot{v}}^n \dot{v}mL + N_{\dot{r}}^n \dot{r}mL^2] - \sum y_i(F_{Xi} + X_{Pi}) \tag{197}$$

The values for a_H follow from a 2D interpolation of the model (see 6.1.2.2). If for a certain course angle ψ^{**}, $a_H(\varepsilon^*, \psi^{**})$ equals zero for all loading angles, a boundary condition is added, so that $x_H(\psi^{**})$ is zero as well. $x_H(\psi^* = \pm 90°)$ is always equal to zero.

6.2 Quadrant 2

No extra analysis is performed for quadrant 2. The added inertia due to propeller action are considered to be zero.

6.3 Quadrant 3

6.3.1 Filter

Tests in the third quadrant are selected:

$$u \leq 0 \tag{198}$$

$$n_1 < 0 \tag{199}$$

$$n_{steered\ 1} < 0 \tag{200}$$

In case of test types MULTI0 and MULTI1, only tests with accelerations equal to zero are selected. Asymmetric conditions are always excluded (see 2.1.1). For bollard pull tests it is assumed that the corresponding yaw and drift angles are equal to 180°.

6.3.2 Regressions

6.3.2.1 Overview

Regression analysis is performed for each DOF, except the longitudinal force. In general the propulsion induced models are tabular in the drift angle β or the yaw angle γ. The input tables can be adapted by the user. As drift and yaw are modelled separately, the following boundary constraint is always applicable:

$$F_{PT}(\gamma = \pm 180°) = 0 \tag{201}$$

The steering induced parameters are constant values. The dependent variable is the measured force minus the modelled hull component and centrifugal terms.

6.3.2.2 Sway force

The regression is performed with the ODRPack program *YRP3.exe*:

$$Y_{RP} = (1 + a_H)\sum F_{Yi} + [Y_{PT}(\beta) + Y_{PT}(\gamma)]\sum T_{Pi} + \frac{n}{n_{MAX}}[Y_{\dot{v}}^{n}\dot{v}m + Y_{\dot{r}}^{n}\dot{r}mL] \tag{202}$$

The following boundary constraints are applied:

$$Y_{\dot{v}}^{n} = 0 \tag{203}$$

6.3.2.3 Heave force

Regression analysis is performed with the ODRPack program *ZRP3.exe*:

$$Z_{RP} = [Z_{PT}(\beta) + Z_{PT}(\gamma)]\sum T_{Pi} \tag{204}$$

6.3.2.4 Roll moment

Regression analysis is performed with the ODRPack program *KRP3.exe*:

$$K_{RP} = -(z_R + a_H z_H T)\sum F_{Yi} + [K_{PT}(\beta) + K_{PT}(\gamma)]T\sum T_{Pi} + \frac{n}{n_{MAX}}[K_{\dot{v}}^{n}\dot{v}mT + K_{\dot{r}}^{n}\dot{r}mT^2] \tag{205}$$

The following boundary constraints are applied:

$$K_{\dot{v}}^{n} = 0 \tag{206}$$

The value for a_H follows from 6.3.2.2, but if $\partial a_H > |a_H|$ a zero value is used instead.

6.3.2.5 Pitch moment

Regression analysis is performed with the ODRPack program *MRP3.exe*:

$$M_{RP} = z_{Hx}L\sum F_{Xi} + [M_{PT}(\beta) + M_{PT}(\gamma)]L\sum T_{Pi} \tag{207}$$

6.3.2.6 Yaw moment

Regression analysis is performed with the ODRPack program *NRP3.exe*:

$$N_{RP} = (x_R + a_H x_H L)\sum F_{Yi} + [N_{PT}(\beta) + N_{PT}(\gamma)]L\sum T_{Pi} + \frac{n}{n_{MAX}}[N_{\dot{v}}^{n}\dot{v}mL + N_{\dot{r}}^{n}\dot{r}mL^2] - \sum y_i(F_{Xi} + X_{Pi}) \tag{208}$$

The following boundary constraints are applied:

$$N_{\dot{v}}^{n} = 0 \tag{209}$$

The value for a_H follows from 6.3.2.2, but if $\partial a_H > |a_H|$ a zero value is used instead.

6.4 Quadrant 4

6.4.1 Filter

Tests in the fourth quadrant are selected:

$$u < 0 \tag{210}$$

$$n_1 > 0 \tag{211}$$

$$n_{steered\ 1} > 0 \tag{212}$$

Only tests of type STATX0 and PMMPSI2 are selected. For the latter an additional criterion applies:

$$|r| \geq 0.015\ rad/s \text{ or } |\dot{r}| \geq 0.001\ rad/s^2 \tag{213}$$

PMMPSI2 tests with these criterion are believed to have negligible oscillations. Asymmetric conditions are always excluded (see 2.1.1). The goal is to further model the average force, as for PMMPSI2 tests no harmonic analysis is available, it is assumed that the measured force equals the average force.

6.4.2 Regressions

6.4.2.1 Overview

Regression analysis is performed for each DOF, except the longitudinal force. In general the propulsion induced models are tabular in the propeller loading angle ε^* (already modelled) or the yaw angle γ. The input tables can be adapted by the user. As propeller loading and yaw are modelled separately, the following boundary constraint is always applicable:

$$F_{PT}(\gamma = \pm 180°) = 0 \tag{214}$$

The steering induced parameters are constant values. The dependent variable is the average force minus the modelled hull component and centrifugal terms and minus the modelled ε^* component for the average force (see 5.3.3).

6.4.2.2 Sway force

The regression is performed with the ODRPack program *YRP4.exe*:

$$Y_{RP} = (1 + a_H)\sum F_{Yi} + Y_{PT}(\gamma)\sum T_{Pi} + \frac{n}{n_{MAX}}[Y_{\dot{v}}^n \dot{v}m + Y_{\dot{r}}^n \dot{r}mL] \tag{215}$$

The following boundary constraints are applied:

$$Y_{\dot{v}}^n = 0 \tag{216}$$

6.4.2.3 Heave force

Regression analysis is performed with the ODRPack program *ZRP4.exe*:

$$Z_{RP} = Z_{PT}(\gamma)\sum T_{Pi} \tag{217}$$

6.4.2.4 Roll moment

Regression analysis is performed with the ODRPack program *KRP4.exe*:

$$K_{RP} = -(z_R + a_H z_H T)\sum F_{Yi} + K_{PT}(\gamma)T\sum T_{Pi} + \frac{n}{n_{MAX}}[K_{\dot{v}}^n \dot{v}mT + K_{\dot{r}}^n \dot{r}mT^2] \tag{218}$$

The following boundary constraints are applied:

$$K_{\dot{v}}^{n} = 0 \quad (219)$$

The value for a_H follows from 6.4.2.2, but if $\partial a_H > |a_H|$ a zero value is used instead.

6.4.2.5 Pitch moment

Regression analysis is performed with the ODRPack program *MRP4.exe*:

$$M_{RP} = z_{Hx} L \sum F_{Xi} + M_{PT}(\gamma) L \sum T_{Pi} \quad (220)$$

6.4.2.6 Yaw moment

Regression analysis is performed with the ODRPack program *NRP4.exe*:

$$N_{RP} = (x_R + a_H x_H L) \sum F_{Yi} + N_{PT}(\gamma) L \sum T_{Pi} + \frac{n}{n_{MAX}} [N_{\dot{v}}^{n} \dot{v} m L + N_{\dot{r}}^{n} \dot{r} m L^2] - \sum y_i (F_{Xi} + X_{Pi}) \quad (221)$$

The following boundary constraints are applied:

$$N_{\dot{v}}^{n} = 0 \quad (222)$$

The value for a_H follows from 6.4.2.2, but if $\partial a_H > |a_H|$ a zero value is used instead.

6.5 Create 2D tables

6.5.1 Overview

In the previous sections regression coefficients were determined per component and per quadrant. The results from the different quadrants are joined together in a 2D table per component:

- Propulsion induced terms, for each DOF, except the longitudinal force, the following tables:
  - $F_{PT}(\varepsilon^*, \beta)$: the average induced force: drift dependency;
  - $F_{PT}(\varepsilon^*, \gamma)$: the average induced force: yaw dependency;
  - $F_{PTA}(\varepsilon^*, \beta)$: amplitude of the oscillations;
  - $\varphi(\varepsilon^*, \beta)$: phase shift of the oscillations;
- Propulsion induced added inertia for the four quadrants;
- Steering induced terms:
  - $a_H(\varepsilon^*, \psi^*)$;
  - $x_H(\varepsilon^*, \psi^*)$;
  - $z_H(\varepsilon^*, \psi^*)$;
  - $z_{Hx}(\varepsilon^*, \psi^*)$.

6.5.2 Determine table keys

In the first place the used input tables for the regression in the different quadrants have to be joined. In the following formulae an array is marked as $[a]_q$, with q the origin of the data, e.g. $3-$: the negative input angles for quadrant 3.

For the propulsion induced terms this is:

- $[\beta] = [\beta]_{3-} + [\beta]_1 + [\beta]_{3+}$;
- $[\gamma] = [\gamma]_{3-} + [\beta]_1 + [\gamma]_{3+}$;
- $[\varepsilon^*] = [-180°, -90°] + [\varepsilon^*]_4 + [90°] + [\varepsilon^*]_2$.

For the steering induced terms this is:

- $[\psi^*] = [-180°] + [\beta]_1 + [180°]$;
- $[\varepsilon^*] = [-180°, -95°, -85°, -2°] + [\varepsilon^*]_1 + [178°] + [180°]$.

6.5.3 Propulsion induced terms

The 2D table of the drift dependency of the average force is created as follows:

- ε^* and β in the third quadrant: the modelled $F_{PT}(\beta)$ value from the third quadrant;
- ε^* and β in the fourth quadrant: the modelled $F_{PT}(\varepsilon^*)$ value from the fourth quadrant;
- ε^* and β in the first quadrant: the modelled $F_{PT}(\beta)$ value from the first quadrant;
- ε^* and β in the second quadrant: the modelled $F_{PT}(\varepsilon^*)$ value from the second quadrant;
- In all the other cases a zero is written.

The 2D table of the yaw dependency of the average force is created as follows:

- ε^* and γ in the third quadrant: the modelled $F_{PT}(\gamma)$ value from the third quadrant;
- ε^* and γ in the fourth quadrant: the modelled $F_{PT}(\gamma)$ value from the fourth quadrant;
- ε^* and γ in the first quadrant: the modelled $F_{PT}(\beta = \gamma)$ value from the first quadrant, but with $F_{PT}(\beta = \gamma = 0) = 0$;
- In all the other cases a zero is written.

The tables of the average force are made symmetric as well. The method depends on the considered force component:

- Heave and pitch:
  - $F'_{PT}(\varepsilon^*, \beta) = \frac{1}{2}[F_{PT}(\varepsilon^*, \beta) + F_{PT}(\varepsilon^*, -\beta)]$;
  - $F'_{PT}(\varepsilon^*, \gamma) = \frac{1}{2}[F_{PT}(\varepsilon^*, \gamma) + F_{PT}(\varepsilon^*, -\gamma)]$;
- Roll, sway and yaw: a split up per quadrant is needed:
  - Quadrants 1 and 3:
    - $F'_{PT}(\varepsilon^*, \beta) = \frac{1}{2}[F_{PT}(\varepsilon^*, \beta) - F_{PT}(\varepsilon^*, -\beta)]$;
    - $F'_{PT}(\varepsilon^*, \gamma) = \frac{1}{2}[F_{PT}(\varepsilon^*, \gamma) - F_{PT}(\varepsilon^*, -\gamma)]$;
  - Quadrant 2:
    - $F'_{PT}(\varepsilon^*, \beta) = \frac{1}{2}[F_{PT}(\varepsilon^*, \beta) + F_{PT}(\varepsilon^*, -\beta)]$;
    - $F'_{PT}(\varepsilon^*, \gamma) = \frac{1}{2}[F_{PT}(\varepsilon^*, \gamma) + F_{PT}(\varepsilon^*, -\gamma)]$;
  - Quadrant 4:
    - $F'_{PT}(\varepsilon^*, \beta) = \frac{1}{2}[F_{PT}(\varepsilon^*, \beta) + F_{PT}(\varepsilon^*, -\beta)]$;
    - $F'_{PT}(\varepsilon^*, \gamma) = \frac{1}{2}[F_{PT}(\varepsilon^*, \gamma) - F_{PT}(\varepsilon^*, -\gamma)]$.

The 2D table of the amplitude of the oscillating force is created as follows:

- ε^* and β in the fourth quadrant: the modelled $F_{PTA}(\varepsilon^*)$ value from the fourth quadrant;
- ε^* and β in the second quadrant: the modelled $F_{PTA}(\varepsilon^*)$ value from the second quadrant;
- In all the other cases a zero is written;
- This table is never made symmetric.

The 2D table of the phase shift of the oscillating force is created as follows:

- ε^* and β in the fourth quadrant: the modelled $\varphi_*(\varepsilon^*)$ value from the fourth quadrant;
- ε^* and β in the second quadrant: the modelled $\varphi_*(\varepsilon^*)$ value from the second quadrant;
- In all the other cases a zero is written;
- This table is never made symmetric.

6.5.4 Steering induced terms

The parameter a_H is a 2D function in the first quadrant, but is constant in the other quadrants (zero in the second quadrant). The 2D table is created as follows:

- ε^* in the third quadrant: the a_H-value from the third quadrant;
- ε^* in the fourth quadrant: the a_H-value from the fourth quadrant;
- ψ^* and ε^* in the first quadrant: $a_H(\varepsilon^*, \psi^*)$ from the first quadrant;
- In all the other cases $a_H = 0$.

The tables for the other terms x_H, z_H and $z_{Hx}$are created as follows:

- ε^* in the third quadrant: the constant value from the third quadrant;
- ε^* in the fourth quadrant: the constant value from the fourth quadrant;
- ψ^* and ε^* in the first quadrant: the course dependent value from the first quadrant, which is repeated for all ε^* in the first quadrant;
- In all the other cases a zero is written.

Symmetric tables are created as well:

- $a'_H(\varepsilon^*, \psi^*) = \frac{1}{2}[a_H(\varepsilon^*, \psi^*) + a_H(\varepsilon^*, -\psi^*)]$

and analogous for x_H, z_H and z_{Hx}.

6.6 Result

6.6.1 Overview

The final computation is to add columns for the modelled propulsion induced forces, steering induced forces and total forces to the datapoint. All the necessary files (the 2D tables created in the previous section) are read to retrieve the model coefficients.

6.6.2 Propulsion induced forces

For the propulsion induced forces, only the average forces are considered in case of oscillations. The model formulations are as follows:

$$Y_P = [Y_{PT}(\varepsilon^*, \beta) + Y_{PT}(\varepsilon^*, \gamma)]\mu_V \sum T_{Pi} + [Y_{\dot{v}}^n \dot{v} m + Y_{\dot{r}}^n \dot{r} m L] \sum \frac{\mu_n n_i}{n_{MAX}} \quad (223)$$

$$Z_P = [Z_{PT}(\beta) + Z_{PT}(\gamma)]\mu_T \sum T_{Pi} \quad (224)$$

$$K_P = [K_{PT}(\beta) + K_{PT}(\gamma)]T \sum T_{Pi} + [K_{\dot{v}}^n \dot{v} m T + K_{\dot{r}}^n \dot{r} m T^2] \sum \frac{\mu_n n_i}{n_{MAX}} \quad (225)$$

$$M_P = [M_{PT}(\beta) + M_{PT}(\gamma)]L \sum T_{Pi} \quad (226)$$

$$N_P = [N_{PT}(\beta) + N_{PT}(\gamma)]L \sum T_{Pi} + [N_{\dot{v}}^n \dot{v} m L + N_{\dot{r}}^n \dot{r} m L^2] \sum \frac{\mu_n n_i}{n_{MAX}} - \sum y_i X_{Pi} \quad (227)$$

In the above equations the following multiplicators were introduced:

- μ_V equals the Froude number in the first quadrant and is equal to one in the other quadrants;
- μ_T equals the Tuck number in the first quadrant and is equal to one in the other quadrants;
- μ_n is equal to 0.5 in a twin propeller setup and equal to one in a single propeller setup, in other words it is assumed that in a twin propeller setup each propeller rate has an equal influence on the propulsion induced added inertia.

Observe that in the first quadrant, with the definition of the course angle ψ^* as in (56) and (57):

$$F_{PT}(\varepsilon^*, \psi^*) \neq F_{PT}(\varepsilon^*, \beta) + F_{PT}(\varepsilon^*, \gamma) \quad (228)$$

however the agreement between measurements and modelled values is acceptable.

6.6.3 Steering induced forces

The model formulations are as follows:

$$X_R = \sum F_{Xi} \tag{229}$$

$$Y_R = \left(1 + a_H(\varepsilon^*, \psi^*)\right) \sum F_{Yi} \tag{230}$$

$$Z_R = 0 \tag{231}$$

$$K_R = -(z_R + a_H(\varepsilon^*, \psi^*) z_H(\varepsilon^*, \psi^*) T) \sum F_{Yi} \tag{232}$$

$$M_R = z_{Hx}(\varepsilon^*, \psi^*) L \sum F_{Xi} \tag{233}$$

$$N_R = (x_R + a_H(\varepsilon^*, \psi^*) x_H(\varepsilon^*, \psi^*) L) \sum F_{Yi} - \sum y_i F_{Xi} \tag{234}$$

6.6.4 Total forces

For each degree of freedom a column is created with the total modelled force:

$$F = F_{IC} + F_{AI} + F_H + F_P + F_R \tag{235}$$

7 Simulation XML

7.1 Vertical response

Once all computations are performed an XML file with coefficients for the ship manoeuvring simulator can be created. However, the determination of the coefficients for the vertical response needed in the simulator are not covered by Regstatx 4.0:

$$Z_{VR} = (Z_{\dot{w}} - 1)m\dot{w} + \left(Z_{\dot{q}}L + x_G\right)m\dot{q} + Z_w\sqrt{\frac{g}{L}}mw + Z_q\sqrt{\frac{g}{L}}mq \tag{236}$$

$$K_{VR} = \left(K_{\dot{p}}mT^2 + K_{\dot{p}u}um\sqrt{\frac{T^3}{g}} - I_{xx}\right)\dot{p} + \left[K_p m\sqrt{gT^3} + K_{up}mTu\left\{-|\varphi|\sqrt{\Delta\overline{GM}_T\left(\left(-K_{\dot{p}}mT^2 + I_{xx}\right)\right)}\right\}\right]p \tag{237}$$

$$M_{VR} = (M_{\dot{w}}mL + mx_G)\dot{w} + \left(M_{\dot{q}}mL^2 - I_{yy}\right)\dot{q} + M_w\sqrt{gL}mw + M_q m\sqrt{gL^3}q \tag{238}$$

These coefficients can be found by numerical means or by the execution of roll, pitch and heave decay tests. In a future version of Regstatx, the determination of the roll vertical response will be included.

The user has to manually add these coefficients, before being able to generate the simulation XML file.

7.2 Selection of coefficient sets

For a given definition, all modelled coefficients are transferred from the SOW XML format to the SIM Fortran XML format. The user has the following options:

- To transfer the original coefficients;
- To transfer the symmetric hull coefficients;
- To transfer the symmetric propulsion and steering (induced) coefficients;
- To transfer the complete set of symmetric coefficients.

The last two options are only advised in case of a twin rudder twin propeller vessel.

8 References

Delefortrie, G.; Eloot, K.; Mostaert, F. (2010). Modelproeven voorspelling manoeuvreergedrag binnenvaart: deelrapport 2. Formulering wiskundige modellering klasse Va-schip bij maximale belading. *WL Rapporten*, 809_01. Waterbouwkundig Laboratorium: Antwerpen

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